POUZDANOST AB MONTAŽNIH VEZA

RELIABILITY OF RC PRECAST JOINT

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1 UVOD

Osnovna funkcija građevinskog inžinjerstva je adekvatno rješenje prihvatanja i provođenja sila kroz sistem (konstrukciju). U tu svrhu se vrši analiza, koja predstavlja idealizaciju konstrukcije u smislu jednostavnijeg, ali logičnog matematičkog rješenja, koje sadrži osnovne elemente stvarne konstrukcije. Kod tradicionalnog pristupa analizi, dejstva na konstrukciju su modelirana kao potpuno definisani, jednostavni geometrijski ili analitički izrazi. Isto tako svojstva materijala su usvojena kao nepromjenljiva unutar konstrukcije i u vremenu. Naravno, dejstva na građevinske konstrukcije nisu nikada u potpunosti poznata u smislu njihovog intenziteta i učestalosti pojave. Usvojene proračunske veličine dejstva su slučajne i svaka konstrukcija može biti izložena dejstvu većeg intenziteta od projektovanog. Savremeni pristup uspostavljanja zavisnosti promjenljivih ulaznih veličina i izlaznih veličina primjenom vještačkih neuronskih mreža prezentiran je u radu [25]. Ovakav pristup je pokazao zadovoljavajuću preciznost u slučajevima kada postoji dovoljan broj izmjerenih eksperimentalnih podataka.

Svi građevinski materijali sadrže mikrokristalne imperfekcije ili lokalne nedostatke. Može se reći da je svaki materijal zbir raznih defekata i za upotrebu se smatra prihvatljivim ako je zbir defekata predvidljiv. Ne postoje, ni do danas, razvijene teorije koje uspostavljaju odnose između čvrstoće i deformacije tijela. Takođe, ne postoje uokvirena istraživanja fenomena kao što su: plastično tečenje i krti lom metala, zamor i puzanje, elastični i postelastični odgovor i sl.

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1 INTRODUCTION

The basic function of building design is adequate acceptance and implementation of loads through the system (construction). For this purpose performs the analysis, which represent of structures idealization in terms of simpler, but the logical mathematical solution, which contains the basic elements of the real structures. With the traditional approach to the analysis, the actions on the structure are modeled as fully defined, simple geometrical or analytical expressions. Also, material properties are adopted as fixed within the structure in time. Of course, the actions on civil structures have never fully known in terms of their intensity and frequency of occurrence. Adopted design values of the actions are random, and each structure can be exposed to the actions with greater values than design values. Modern approach to establishing the dependence of variable input and output values using artificial neural networks is presented in the paper [25]. This approach has proved sufficiently precise in cases where a sufficient number of measured experimental data.

All building materials contain microcrystalline imperfections or local defects. It can be said that each material is sum of various defects and for the use is considered acceptable if the sum of defects are predictable.

Not today, developed theories that relate the strength and deformation of the body not exist. Also, there are no framed research of phenomena such as plastic creep and brittle fracture, fatigue, and elastic and post-elastic response etc.

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Načini na koje dolazi do otkaza građevinskih konstrukcija, njihova učestalost, ekonomske i socijalne posljedice, pokazuju priličnu razliku između hipotetičkih ii realnih konstrukcija. Proračunsko opterećenje, uslovi na gradilištu, svojstva materijala, razvijene proračunske procedure i adekvatnost predviđenih veličina i oblika konstrukcije i njenih elemenata je ponekad daleko od stvarnih parametara. Navedeni parametri su rezultat kompleksnih međusobnih odnosa, defekata materijala, strukturalnih odstupanja, ljudskih grešaka, i imaju karakter slučajno promjenljivih. Stoga je za kvantifikaciju sposobnosti konstrukcije da ispuni očekivane zahtjeve neophodno određivanje mjere uspješnosti, koja se naziva **pouzdanost**.

Uobičajena definicija pouzdanosti je:

Pouzdanost je vjerovatnoća da će objekat (sistem) adekvatno ispuniti njegovu zahtijevanu funkciju, u definisanom vremenskom periodu, pod određenim okolnostima.

Prvi korak u ocjenama konstrukcije korištenjem teorije pouzdanosti jeste pribavljanje svih neophodnih ulaznih podataka koji opisuju konstrukciju. Analiza pouzdanosti započinje sa identifikacijom različitih izvora rizika na konstrukciji. U radu [21] prezentirano je stablo slučaja, koje razmatra sve moguće uzročnike otkaza elementa i podsistema koji vodi otkazu sistema. Za svku komponentu u stablu slučaja mora se definisati granično stanje. Analiza pouzdanosti uklljučuje proračun i predviđanje vjerovatnoće narušavanja graničnog stanja u bilo kojem trenutku eksploatacionog vijeka konstrukcije. Vjerovatnoća pojave događaja kao što je narušavanje graničnog stanja je numerička vrijednost mogućnosti pojave. Iz jednačine graničnog stanja može se za svki parametar proračunati vjerovatnoća otkaza p_f i indeks sigurnosti β. Kada se odredi vjerovatnoća, slijedeći korak je odabir alternative proračuna koja poboljšava pouzdanost konstrukcije, a minimizira rizik od otkaza. Pri tome je odgovarajuće opažanje nepouzdanosti ključno za sigurnost i efikasnu odluku. Probabilistička proračunska procedura je analiza konstrukcije koja uzima u obzir informacije probabilističkog karaktera o kapacitetu I zahtjevima. Ovakav pristup uslovljava sveobuhvatnu analizu konstrukcije sa svrhom optimaliziranja zahtjeva sigurnosti i ekonomičnosti.

Koncept pouzdanosti je danas opšte prihvaćen za projektovanje konstrukcija, iako je dobro poznato da je često potrebno značajno pojednostavljenje proračunskog problema da bi se ovaj koncept efikasno primijenio. Ovo je prije svega iz slijedeća dva razloga:

(1) U svojoj najjednostavnijoj formulaciji procedure zasnovane na pouzdanosti zahtjevu prikaz performansi konstrukcije u eksplicitnim relacijama između varijabli (promjenljivih) dejstava i otpornosti. Ali kada je ponašanje konstrukcije uslovljeno nelinearnostima više varijabli, kao što je uvijek slučaj sa betonom, ovakve relacije općenito su dostupne samo u implicitnom obliku.

(2) Kod konstruktivnih sistema sa više komponenti, potpuna analiza pouzdanosti podrazumjeva analizu pouzdanosti dijelova sistema i sistema. Zavisno od broja i rasporeda dijelova ocjena pouzdanosti sistema može postati vrlo komplikovana pa čak i praktično nemoguća za velike konstruktivne sisteme.

Ways to come to the failure of civil structures, their frequency, economic and social consequences, show a reasonable difference between the hypothetical and real structures. Design actions, site conditions, material properties, developed design procedures and the adequacy of the anticipated size and form of structures, and its elements, are sometime far from the real parameters. The mentioned parameters are the result of complex mutual relations, material defects, structural differences, human errors, and have the character of random variable. Therefore, for the quantification of structural capacity to meet expected requirements it is necessary to define measures of success, which is called **reliability**.

The usual definition of reliability is:

Reliability is the probability that the building (system) is adequate to meet its desired function in the defined time period, under certain circumstances.

The first step in reliability-based assessment is to acquire all necessary input data describing the structure. The reliability analysis starts with the indentification of the different sources of risk of the structure. In the paper [21] is presented fault tree, which regards all possible causal sequences of component and subsystem failures that lead to system failure. For all components in a fault tree a limit-state has to be defined. Reliability analysis includes the calculation and prediction of the probability of limit-state violation at any stage during a structure's life. The probability of the occurrence of an event such as a limit-state violation is a numerical measure of the chance of its occurring. From the limit state equation can be calculated for each parameter probability of failure p_f and safety index β. When the probability determined, the next goal is to choose design alternatives that improve structural reliability and minimize the risk of failure. Where appropriate perceptions of uncertainty are essential for safe and efficient decisions. Probabilistic design procedures is the analysis of structures which take into account probabilistic-based information about capacities and demands. This provides a comprehensive analysis of structures with the aim of optimizing the requirements of safety and economy.

Reliability-based concepts are nowadays widely accepted in structural design, even if it is well known that, before such concepts can be effectively implemented, the actual design problem often needs to be considerably simplified. This is mainly due to the two following reasons:

(1) In their simplest formulation, reliability-based procedures require the structural performance to be represented by explicit functional relationships among the load and the resistance variables. But, unfortunately, when the structural behavior is affected by several sources of non-linearity, as always happens for concrete structures, such relationships are generally available only in an implicit form.

(2) For structural systems with several components, a complete reliability analysis includes both component-level and system-level estimates. Depending on the number and on the arrangement of the components, system reliability evaluations can become very complicated and even practically impossible for large structural systems.

2 OSNOVE TEORIJE VJEROVATNOĆE I MJERODAVNI STATISTIČKI PARAMETRI

Vjerovatnoća određenog događaja u trenutku vremena T se definiše izrazom:

2 BASIS OF THE THEORY OF PROBABILITY AND THE RELEVANT STATISTICAL PARAMETERS

The probability of particular events in the moment of time T is defined expression:

$$P(T \le t) = F(t); \qquad t \ge 0 \tag{1}$$

Vrijednost vjerovatnoće može biti u granicama $0 \le P(A) \le 1$. Prema tome, sigurnost se može definisati kao $P[C] = 1{,}0$. Isto tako mora vrijediti jednakost:

The value of probability can be inside $0 \le P(A) \le 1$. Accordingly, security can be defined as $P[C] = 1{,}0$. Also must be valid equality:

$$P[success + failure] = 1 \quad \Rightarrow \quad P[success] + P[failure] = 1 \tag{2}$$

Vjerovatnoća uspješnosti konstrukcije je njena pouzdanost i može se definisati izrazom:

The probability of structure success is its reliability and can be defined by expression:

$$R + P(f) = 1 \tag{3}$$

Uvrštavanjem (1) u (3) dobije se funkcija pouzdanosti:

Substituting (1) to (3) obtain the reliability function:

$$R(t) = 1 - F(t) = P(T > t) \tag{4}$$

F(t) je funkcija raspodjele otkaza, koja pokazuje vjerovatnoću otkaza sistema do trenutka vremena t. F(t) se još zove kumulativna funkcija raspodjele. Derivacijom funkcije raspodjele otkaza dobije se funkcija gustine otkaza,

F (t) is a function of the failure distribution, which quantify probability of system failure until moment of time t. The function F (t) is also called cumulative distribution function. With derivation of function of failure distribution obtain the function of failure density.

$$f(t) = \frac{dF(t)}{dt} \tag{5}$$

Prema tome izraz (4) dobija oblik:

Thus expression (4) gets the form:

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt = \int_t^\infty f(t)dt \tag{6}$$

Promjena intenziteta otkaza u eksploeatacionom vijeku sistema (konstrukcije) može se iskazati pomoću funkcije intenziteta otkaza date izrazom [19]:

Change of failure intensity during system (structure) service life can be expressed using a function of failure intensity expressed as [19]:

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{7}$$

Funkcija pouzdanosti se može izraziti preko funkcije intenziteta otkaza u obliku:

Reliability function can be expressed through the functions of the failure intensity with form:

$$R(t) = e^{-\int_0^t \lambda(t)dt} \tag{8}$$

Ključni statistički parametri diskretizovane raspodjele slučajno promjenljivih, očekivana vrijednost i varijacija, definišu se iz analogije krutog štapa opterećenog nizom vertikalnih koncentričnih sila f_i na udaljenostima x_i, kako je pokazano na slici 1.

Iz uslova ravnoteže je poznato da je sila koja uravnotežuje vertikalne sile f_i jednaka

Key statistical parameters discrete distribution of random variable, expected value and variation, can be defined by analogy from solid beam loaded with a series of concentric vertical forces f_i at distances x_i, as shown in Fig. 1.

From the condition of equilibrium is well known that the force that balances the vertical forces f_i equal to,

$$M = \sum_{i=1}^{n} f_i \tag{9}$$

dok se njen položaj određuje izrazom,

while its position is determined expression,

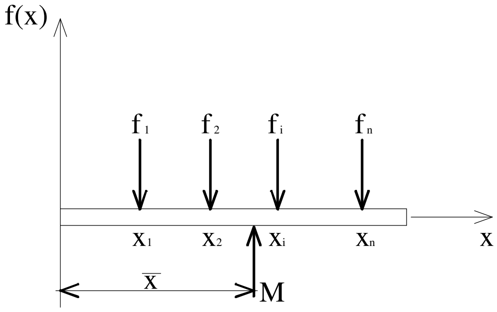
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{M} \tag{10}$$

Ako se pretpostavi da su diskretne koncentrisane sile sa slike 1 vjerovatnoće mogućih događaja $x_1, x_2, ..., x_n$ i

If we assume that the discrete concentric force (Fig.1) are probability of possible events $x_1, x_2, ..., x_n$ and vrijedi jednačina (2) (*M=1*), izraz (9) se može napisati u obliku:

equation (2) is valid (M = 1), expression (9) can be rewritten in the form:

$$E[x] = \bar{x} = \sum_{i=1}^{n} x_i f_i \tag{11}$$



Sl.1. Koncentisane sile na krutoj gredi
Fig.1. Concentric forces on solid beam

Mjera varijabilnosti (rasipanja) slučajnih varijabli se definiše takođe iz statike, analogijom sa momentom inercije,

$$I_y = \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 f_i \tag{12}$$

Analogy with the moment of inertia can be defined measure of variability (dispersion) of random variables

$$V[x_i] = \sum_{i=1}^{N} \left(x_i - \bar{x}\right)^2 f_i \tag{13}$$

Na osnovu izraza (10) i (11) varijanca se može izraziti kao očekivanje,

On the basis of expression (10) and (11) the variance can be expressed as an expectation,

$$V[x_i] = E\left[x_i^{\,2}\right] - \left(E[x_i]\right)^2 \tag{14}$$

Standardna devijacija je: $\sigma[x_i] = \sqrt{V[x_i]}$

Standard deviation is: $\sigma[x_i] = \sqrt{V[x_i]}$

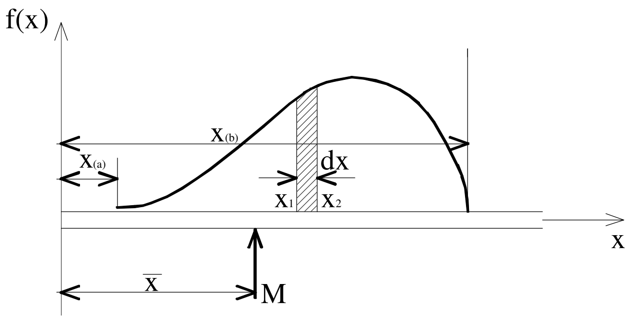
Drugi važan parametar rasipanja rezultata je koeficijent varijacije, koji predstavlja mjeru odstupanja od centralne osi,

Another important parameter of random variables scattering is the coefficient of variation, which is a measure of deviation from the central axis,

$$V(x) = \frac{\sigma(x)}{E(x)} \cdot 100\ (\%) \tag{15}$$

Za analizu kontinuirane raspodjele slučajno promjenljive može se primjeniti analogija sa jedankopodijeljenim vertikalnim opterećenjem na krutoj gredi od *x(a)* do *x(b)* [14], prikazano na slici 2.

For the analysis of continuous distribution of random variable can be applied analogy with distributed vertical load on solid beam, from *x(a)* to *x(b)* [14], presented in Fig.2.



Sl.2 - Jenoliko podijeljeno vertikalno opterećenje na krutoj gredi
Fig.2 - Continuous distribution of vertical load on solid beam

U ovom slučaju, izraz (9) je oblika,

In this case, expression (9) has the form,

$$M = \int_{x(a)}^{x(b)} f(x)\,dx \tag{16}$$

a izraz (10),

and expression (10),

$$\bar{x} = \frac{\int_{x(a)}^{x(b)} x f(x) dx}{M} \quad (17)$$

Površina ispod krive $f(x)$ za interval dx je vjerovatnoća da će vrijednost x biti u intervalu od x_1 do x_2:

Area under the curve $f(x)$ for interval dx is the probability that the value of x will be in the interval from x_1 to x_2:

$$\int_{x_1}^{x_2} f(x) dx = P[x_1 \le x \le x_2] \quad (18)$$

Površina ispod krive u intervalu $x(a)$ do $x(b)$ mora biti 1 (jedan).

Area under the curve in the interval of $x(a)$ to $x(b)$ must be 1 (one).

$$\int_{x_{(a)}}^{x_{(b)}} f(x) dx = 1 \quad (19)$$

Za kontinuiranu raspodjelu očekivana vrijednost je,

For continuous distribution expected value is

$$E[x] = \int_{x(a)}^{x(b)} x f(x) dx \quad (20)$$

a varijanca,

and variance,

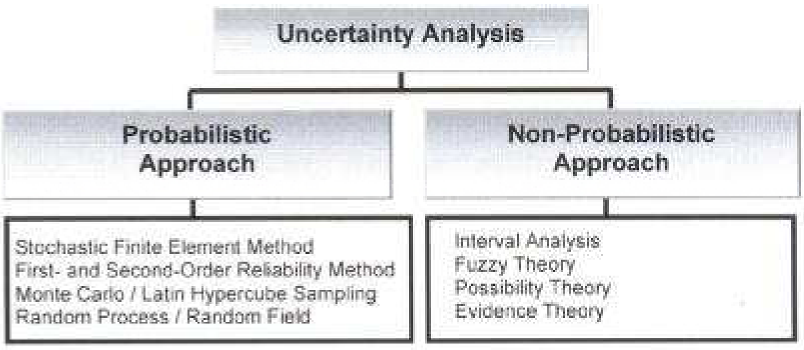
$$V[x] = \int_{x(a)}^{x(b)} (x - \bar{x})^2 f(x) dx \quad (21)$$

3 FUNKCIJA GUSTOĆE VJEROVATNOĆE

Teorija vjerovatnoće razmatra vjerovatnoću određene pojavnosti događaja i kvantificira mjere nesigurnosti slučajnih događaja. Metodologija, kojom se mogu razmotriti slučajnosti ili nesigurnosti u podacima ili modelima, poznata je kao analiza nesigurnosti ili stohastička analiza. Na slici 3 pokazane su razne metode analize nesigurnosti zasnovane na prikazu nesigurnosti.

3 PROBABILITY DENSITY FUNCTION

Probability theory treats the likelihood of a given event's occurrence and quantifies uncertain measures of random events. Methodologies, which can consider the randomness or uncertainty in the data or model, are known as uncertainty analysis or stochastic analysis. Figure 3 shows various methods of uncertainty analysis based on the representation of uncertainties.



Sl.3 - Kategorije analize nepouzdanosti[7]
Fig.3 - Uncertainty Analysis Categories[7]

Probabilistički pristup zasnovan je na teoretskim osnovama funkcije gustoće vjerovatnoće i uvođenju koriš'enja slučajno promjenljivih za prikaz nesigurnosti, dok deterministički (ne-probabilistički) pristup koristi neprecizno znanje o stvarnim vrijednostima parametara.

Funkcija gustoće vjerovatnoće reprezentira relativnu učestalost određene realizacije slučajne promjenljive.

The probabilistic approach is based on the theoretical foundation of the probability density function information and introduces the use of random variables to represent uncertainty, while the non-probabilistic approach manages imprecise knowledge about the true value of parameters.

The probability density function represents the

Osnova statističke obrade podataka jeste definisanje zakonitosti (funkcije) raspodjele slučajno promjenljivih. Podaci utvrđeni analizom rezultata eksperimenata imaju određenu zakonitost raspodjele. U postupku analize raspodjela utvrđenih podataka se upoređuje sa poznatim teorijskim zakonitostima raspodjele. Izbor najpovoljnije zakonitosti raspodjele započinje postavljanjem hipoteze o mogućem teorijskom zakonu raspodjele. Ocjena postavljene hipoteze se vrši preko statističkih testova zasnovanih na maksimalno dozvoljenom odstupanju empirijskog i teorijskog zakona raspodjele. Aproksimacija eksperimentima utvrđene raspodjele nekom poznatom teorijskom raspodjelom omogućava korištenje analize poznate teorijske raspodjele, što značajno pojednostavljuje postupak ocjene pouzdanosti sistema (konstrukcije).

Nekoliko kontinuiranih raspodjela igra značajnu ulogu u građevinarstvu. Najvažnije su jednolika, eksponencijalna, gama, beta, Weibull-ova, binomna, lognormalna i normalna raspodjela. Najjednostavniji oblik kontinuirane raspodjele je jednolika sa funkcijom gustoće vjerovatnoće, srednjom vrijednosti i varijancom,

$$f(x)=\frac{1}{b-a},\ a\le x\le b \qquad (22)$$

$$E[x]=\frac{b+a}{2} \qquad (23)$$

Kod eksponencijalne raspodjele vjerovatnoća da se neće desiti slučajno promjenljiva *X* u vremenskom intervalu *t* je,

relative frequency of certain realization for random variables. The basis of statistical data processing is to define the probability density function. Data determined during analysis of the experiments results have particular laws of distribution. In the process of analysis the distribution of determined data comparisons with known theoretical laws of distribution. Selecting the best laws of distribution begins by setting the hypotheses about the possible theoretical distribution law. Evaluation of set hypothesis is done by statistical tests based on the maximum permitted deviations of the empirical and theoretical laws of distribution. Approximation of the distribution determined by experiments with a known theoretical distribution analysis allows the use of well-known theoretical distribution, which greatly simplifies the process of assessing the reliability of the system (structure).

Several continuous distribution play useful roles in civil engineering. The more important ones are the Uniform, Exponential, Gamma, Beta, Weibull, Binom, Lognormal and Normal distribution.

The simplest type of continuous distribution is the uniform with probability distribution function, mean and variance

$$V[x]=\frac{(b-a)^2}{12} \qquad (24)$$

By exponential distribution the probability of no occurrences of the random variable *X* during a time interval *t* is

$$P(X=0)=e^{-\lambda t} \qquad (25)$$

a kumulativna funkcija raspodjele je,

and cumulative distribution function is

$$P(T\le t)=F_T(t)=1-e^{-\lambda t} \qquad (26)$$

Ova raspodjela je primjenljiva za elemente koji ne mijenjaju svoje karakteristike u veoma dugom eksploatacionom vijeku. Pogodna je za betonske konstrukcije.

Sa fizikalnog stanovišta, inžinjeri i naučnici su ustanovili da empirijske raspodjele mnogih prirodnih procesa i procesa u konstrukciji približno odgovaraju gama raspodjeli. Standardna gama funkcija raspodjele vjerovatnoće može se napisati,

This distribution is applicable for elements that do not change their characteristics in a very long period of exploitation. It is suitable for concrete structures.

From the physical viewpoint, engineers and scientists have found that the empirical distributions of many natural and structural processes closely resemble the gamma. The standard gamma probability distribution function is written as

$$f(t)=\begin{cases}\dfrac{t^{r-1}e^{-t}}{\Gamma(r)} & for\ 0\le t\ and\ r>0\\ 0\ otherwise\end{cases} \qquad (27)$$

$\Gamma(r)$ je standardna gama funkcija definirana kao,

$\Gamma(r)$ is complete (standard) gamma function defined as

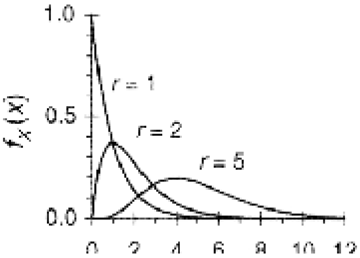
$$\Gamma(r)=\int_0^\infty t^{r-1}e^{-t}dt,\ for\ r>0$$
$$=0\ otherwise \qquad (28)$$

Parametar *r* je poznat kao parametar oblika.

Beta raspodjela se koristi kod eksperimenata koji se ponavljaju *N* puta sa odvojenim izlazima za svaki eksperiment, gdje je *x* broj uspješnih eksperimenata. Modeli sa beta raspodjelom imaju posebnu ulogu kod metoda odlučivanja. Raspodjela slučajno promjenljive koja poprima vrijednosti u određenom intervalu. Beta funkcija raspodjele vjerovatnoće data je izrazom,

The parameter *r* is known as the shape parameter.

The beta distribution is used in the experiments repeated *N* times with independent output for each experiment, where x is the number of successful experiments. The beta distribution models plays a special role in decision methods. The distribution of random variable that takes value in the interval. The beta probability distribution function is given by

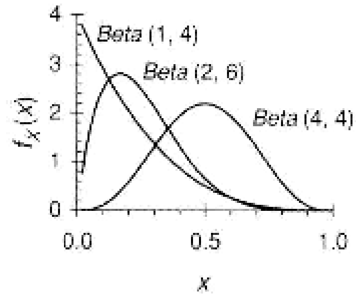


Sl.4 - Standardna gamma funkcija gustoće vjerovatnoće [16]
Fig.4 - Standard gamma probability density functions [16]

$$f(x|\alpha,\beta)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1},\ for\ 0<x<1,\alpha>0,\beta>0$$
$$=0\quad otherwise \tag{29}$$

sa beta funkcijom, with beta function

$$B(\alpha,\beta)=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx \tag{30}$$

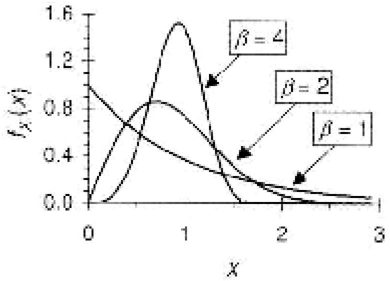


Sl.5 - Standardna beta funkcija gustoće vjerovatnoće [16]
Fig.5 - Standard beta probability distribution functions [16]

Weibull-ova raspodjela se koristi u praksi za sisteme koji pokazuju porast tendencije ka otkazu u vremenu. Weibull-ova funkcija raspodjele vjerovatnoće data je izrazom,

Weibull distribution is used in practice for systems that show a growing tendency to failure over time. The Weibull probability distribution function is given by

$$f(x)=\frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1}\exp\left[-\left(\frac{x}{\lambda}\right)^{\beta}\right],\ for\ x>0,\lambda>0$$
$$=0\quad otherwise \tag{31}$$



Sl.6 - Weibull-ova funkcija gustoće vjerovatnoće [16]
Fig.6 - Weibull probability distribution functions [16]

Dok se duktilni otkaz armiranobetonskog konstruktivnog elementa dešava simultano duž površine otkaza, okarakterisan Gauss-ovom raspodjelom čvrstoće kon-

While ductile failure of reinforced concrete structure elements occurs simultaneously along the failure surface and is characterized by Gaussian distribution of strukcije, bez uzimanja efekta veličine elementa, na kvazikrti otkaz značajno utiče veličina elementa i za velike elemente se pojavljuju ekstremne statističke vrijednosti po modelu najslabije karike, što vodi, prema [2], do Weibull-ve raspodjele čvrstoće konstrukcije (uz pretpostavku da se otkaz dešava sa početkom mikro prslina).

Lognormalna raspodjela je vrlo dobar model za proučavanje otkaza usljed zamora i njegove značajne primjene u području održavanja. Lognormalna funkcija raspodjele vjerovatnoće data je izrazom,

structural strength with no size effect, quasibrittle failures propagates, exhibits a strong size effect and, at large sizes, follows extreme value statistics of the weakest-link chain model, which leads, according to [2], to Weibull distribution of structural strength (provided that failure occurs at macro-crack initiation).

Lognormal distribution is a very good model for studying the failure of which is the cause of fatigue and has significant application in the field of maintenance. The Lognormal probability distribution function is given by

$$f(x)=\frac{1}{\sigma\cdot x\cdot\sqrt{2\pi}}e^{\left[\frac{-(\ln x-\lambda)^2}{2\sigma^2}\right]} \tag{32}$$

Normalna funkcija raspodjele vjerovatnoće data je izrazom,

The Normal probability distribution function is given by

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\left[\frac{-(x-\bar{x})^2}{2\sigma^2}\right]} \tag{33}$$

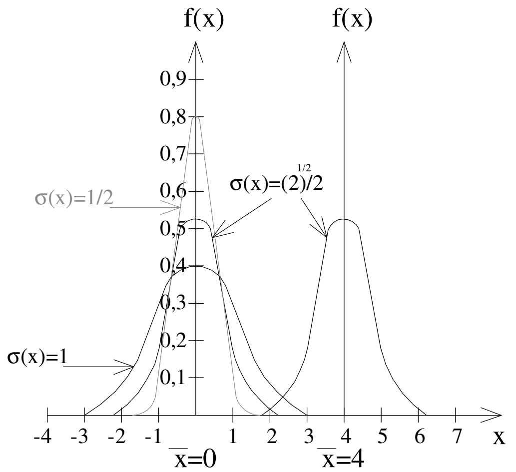
U radovima [7], [14], [16], [19], [20] i [22] opisane su detaljnije funkcije raspodjele slučajno promjenljive, koje se koriste za analizu pouzdanosti tehničkih sistema.

Za analizu pouzdanosti građevinskih konstrukcija obično se koristi normalna raspodjela vjerovatnoće u obliku,

In the papers [7], [14], [16], [19], [20], and [22] are described in some detail the functions of distribution of random variable, which is used to analyze the reliability of technical systems.

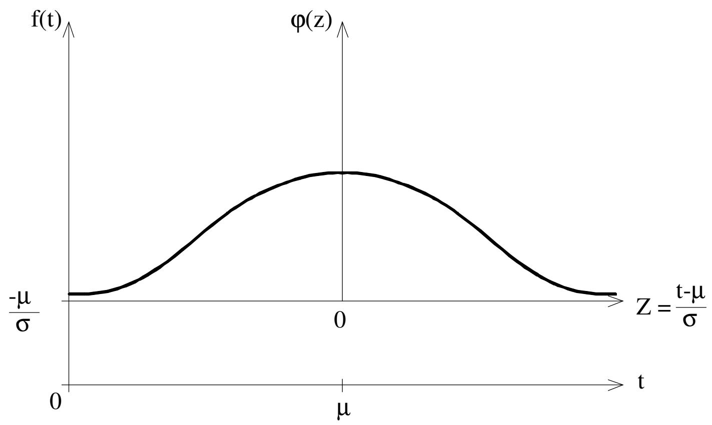
For reliability analysis of civil structures commonly used normal probability distribution in form

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{34}$$



Sl.7 - Neki oblici krive normalne raspodjele

Fig.7 - Some forms of normal distribution curves



Sl.8 – Funkcija gustoće vjerovatnoće za standardnu normalnu raspodjelu

Fig.8 – Probability density function for standardized normal distribution

Kumulativna funkcija normalne raspodjele može se ocijeniti samo numeričkim metodama. U svrhu evaluacije u praksi se koristi standardizovana kriva sa transformacijom promjenljive *X* u *Z*:

$$z=\frac{t-\mu}{\sigma} \tag{35}$$

Cumulative distribution function of normal distribution can only be evaluated by numerical methods. In practice one uses the standardized curve for the purpose of evaluation with the transformation of the variate *X* to *Z* as follows:

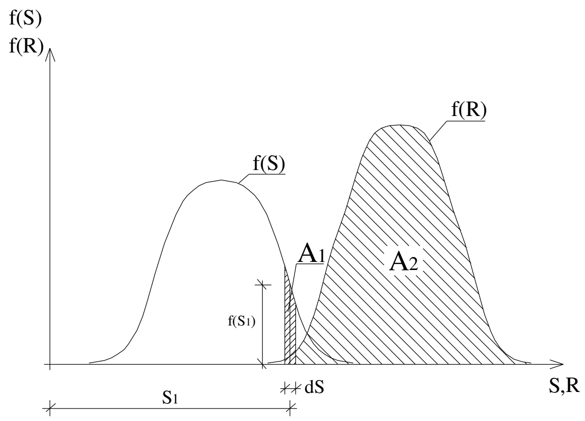
$$\varphi(z)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}z^2}\;;\quad -\infty<z<\infty \tag{36}$$

4 MJERE POUZDANOSTI

Normativi po kojima se vrši analiza nosivosti građevinskih konstrukcija zasnovani su na determinističkim i semi-probabilističkim proračunskim procedurama, sa usvojenim faktorima sigurnosti. Odabrani faktori sigurnosti za pojedine proračunske situacije su rezultat prethodno provedene analize uticajnih parametara primjenom teorije pouzdanosti.

4 MEASURES OF RELIABILITY

Standards in use to perform analysis of civil structures capacity are based on deterministic and semi-probabilistic design procedures, with the adopted safety factors. Selected safety factors for the individual design situations are the result of a previously performed analysis of influential parameters using the theory of reliability.



Sl.9 – Definicija pouzdanosti
Fig.9 – Definiton of reliability

Vjerovatnoća da je uticaj S_1 u intervalu *ds* jednaka je površini A_1, , i može biti izražena kao,

Probability that the action S_1 is in the interval *ds,* equal to the area A_1, , can be expressed:

$$P\left(S_1 - \frac{ds}{2} \le s \le S_1 + \frac{ds}{2}\right) = f(S_1)ds = A_1 \tag{37}$$

Vjerovatnoća da je otpornost veća od uticaja S_1 jednaka je površini A_2:

Probability that the resistance is greater than the action S_1 is equal to the area A_2:

$$P(R - S_1) = \int_{s_1}^{\infty} f(R)dR = A_2 \tag{38}$$

Pouzdanost je proizvod vjerovatnoće (37) i (38),

Reliability is the product of probability (37) and (38),

$$dR = f(S_1)dS \int_{s_1}^{\infty} f(R)dR \tag{39}$$

odnosno opšti izraz za pouzdanost je oblika:

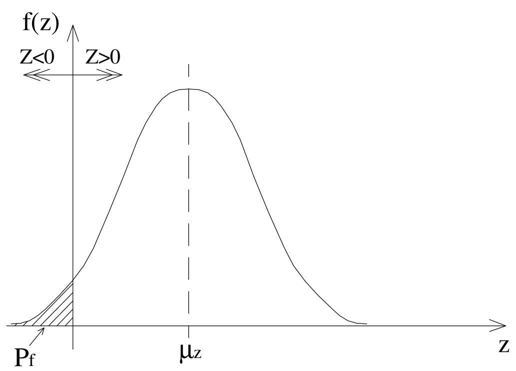
and the general expression for the reliability is:

$$R = \int dR = \int_{-\infty}^{\infty} f(S)\left[\int_{s}^{\infty} f(R)dR\right]dS \quad \text{ili} \quad R = \int dR = \int_{-\infty}^{\infty} f(R)\left[\int_{-\infty}^{R} f(S)dS\right]dR \tag{40}$$

Uvođenjem smjene $\xi = R - S$ ($\xi > 0$) i standardizirane slučajno promjenljive $Z = \dfrac{\xi - \bar{\xi}}{\sigma_\xi}$ funkcija pouzdanosti ima oblik:

With the introduction of $\xi = R - S$ ($\xi > 0$) and standardized random variable $Z = \dfrac{\xi - \bar{\xi}}{\sigma_\xi}$ reliability function has the form:

$$R = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\frac{\bar{\xi}}{\sigma_\xi}}^{\infty} e^{-\frac{1}{2}Z^2} dZ \tag{41}$$



Sl.10 – Standardizirana funkcija gustoće vjerovatnoće
Fig.10 – Standardized probability density function

Neadekvatnost sistema da ispuni zahtjeve, mjerena kao p_f, vezana je sa dijelom raspodjele gdje mjera sigurnosti Z poprima negativne vrijednosti, a odovarajuća pouzdanost je,

The inadequacy of the system to meet the demand, as measured by p_f, is associated with that portion of the distribution of the safety margin wherein Z takes negative values, and corresponding reliability is

$$R = 1 - p_f \tag{42}$$

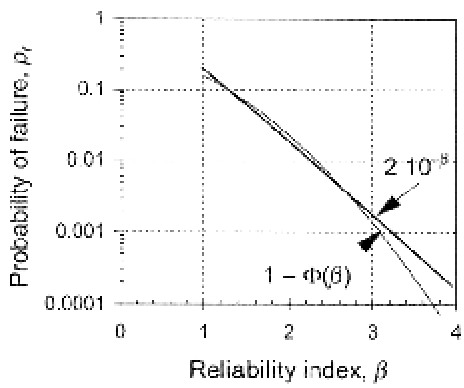
Važna mjera adekvatnosti inžinjerskog projektovanja je indeks pouzdanosti, definisan kao odnos između srednje vrijednosti i standardne devijacije mjere sigurnosti sistema.

An important measure of the adequacy of an engineering design is the reliability index, defined as the ratio between the mean and standard deviation of the safety margin of the system.

$$\beta = \frac{\mu_z}{\sigma_z} \tag{43}$$

Opšti izraz za indeks pouzdanosti u odnosu na prva dva momenta kapaciteta i funkciju zahtjeva može se napisati kao,

A general expression for the reliability index in term of the first two moments of the capacity and the demand function can be written as



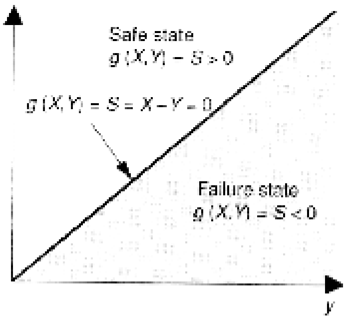
Sl.11 – Snaga aproksimacije vjerovatnoće otkaza kao funkcija indeksa pouzdanosti za normalnu raspodjelu S i R [16]
Fig.11 – Power approximation of the probability of failure as a function of the reliability index for normally distributed S and R [16]

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^{\,2} - 2\rho_{RS}\sigma_R\sigma_S + \sigma_S^2}} \tag{44}$$

Ocjena pouzdanosti podrazumjeva usporedbu proračunatog indeksa pouzdanosti β sa vrijednosti indeksa pouzdanosti koji se smatra adekvatnim na osnovu prethodnih iskustava na datom sistemu. U tom smislu, mora se uspostaviti odnos između kapaciteta sistema i zahtjeva (uticaji). Ukoliko su kapacitet i uticaji jednaki sistem je u graničnom stanju, koje se može pisati u obliku,

The assessment of reliability mean comparing the calculated reliability index β with that found to be adequate from previous experience for the given system. For this purpose, one must establish a relationship between the capacity of the system and the demand (load). If the capacity and load equal, this is a limit state, which can be written in the form

$$g(R,S) = R - S = 0 \tag{45}$$



Sl.12 – Stanje otkaza, stanje sigurnosti i granično stanje [20]
Fig.12 – Failure state, safe state and limit state [20]

Općenito su u građevinarstvu svi problemi funkcija više slučajno promjenljivih, gdje je važno da sve slučajno promjenljive imaju odgovarajuću pouzdanost. Pouzdanost sistema slučajno promjenljivih, kod kojeg svaki element mora funkcionisati da bi sistem funkcionisao predstavlja se kao serijski spoj uticajnih parametara [19], [22]:

Generally in the civil engineering all the problems are function of several random variable, where it is important that all random variables have adequate reliability. Reliability of random variable system, where each element must function that the system functioned, present as a serial combination of influential parameters [19], [22]:

$$R = 1 - P\left(\bar{x}_1 \cup \bar{x}_2 \cup .... \cup \bar{x}_n\right) \tag{46}$$

i generalno je sistem međusobno nezavisnih slučajno promjenljivih, za koje vrijedi izraz,

and generally is a system of mutually independent random variable, for which valid expression:

$$R = P(x_1)P(x_2)....P(x_n) = \prod_{i=1}^{i=n} P(x_i) \tag{47}$$

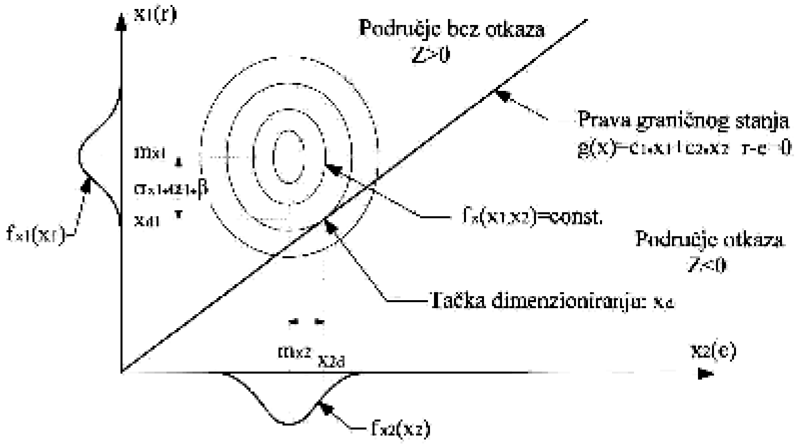
Za *m* nezavisnih slučajno promjenljivih x_i, sa normalnom raspodjelom, dobije se *m*-dimenzionalna raspodjela gustoće prema multiplikacionom pravilu proračuna vjerovatnoće [12]:

For *m* independent random variable x_i, with normal distribution, the result is m-dimensional density distribution according to multiplication rule of probability calculation [12]:

$$f_x(x_1, x_2, ..., x_m) = f_{x1}(x_1) \cdot f_{x2}(x_2) \cdot .. \cdot f_{xm}(x_m) = \frac{1}{(2\pi)^{\frac{m}{2}} \cdot \prod_{i=1}^{m} \sigma_{x_i}} \cdot \exp\left(-\frac{1}{2} \cdot \sum_{i=1}^{m} \left(\frac{x_i - m_{x_i}}{\sigma_{x_i}}\right)^2\right) \tag{48}$$

U slučaju dvije osnovne promjenljive: x_1 i x_2, odnosno *R* (otpornost) and *S* (dejstva), dobiju se hiper površine, istih raspodjela gustoće, odnosno m-dimenzionalne kugle [12] (Slika 10).

In the case of two basic variables: x_1 and x_2, and *R* (resistance) and *S* (action), get a hyper surface, the same distribution density, and m-dimensional balls [12] (Fig.10).



Sl.13 - Jednačina graničnog stanja u x domenu [12]
Fig.13 - Limit state equation in x – area [12]

U praksi funkcija gustoće nije poznata i najviše što je poznato je set *n* slučajno promjenljivih *X* koje definiraju problem konstrukcije (kao što su mehaničke i geometrijske karakteristike, pokretna i stalna dejstva, dejstvo prednaprezanja itd.). Štaviše, kod proračuna betonskih konstrukcija granična stanja su obično formulisana preko funkcija slučajno promjenljivih *Y=Y(X)* koje opisuju odgovor konstrukcije (odnosno naprezanja, dilatacije itd.), i ovakvi izvodi su generalno dostupni samo u implicitnom obliku. Stoga je neophodan numerički pristup. Za rješavanje sistema slučajno promjenljivih razvijene su 3 metode rješavanja:

1. **Tačna metoda**, koja zahtjeva kao ulazne podatke poznate funkcije distribucije vjerovatnoće za sve promjenjive. U postupku rješavanja primjenjuje se numerička integracija i Monte Carlo metoda. Ovaj postupak zahtjeva opsežnu kompjutersku analizu.
2. **FOSM** (First order second moment method), gdje se u postupku rješavanja koriste Taylor-ovi redovi.
3. **PEM** (Point estimate method) koja se najviše koristi u analizama jer omogućava postepeni razvoj rješenja.

Analitička integracija je moguća samo kod specijalnih slučajeva. Numeričko rješenje problema je jednostavnije, ali ako je pod integralom više od dvije slučajno promjenljive, numerička integracija ne može u svim slučajevima dati dovoljno tačno rješenje. Sa povećanjem broja varijabli značajno rastu proračunski zahtjevi, jer su u tim slučajevima integraciona područja složenije geometrijske figure, n-dimenzionalne sfere. Stoga su za određene metode razvijena rješenja na osnovu numeričke integracije, kao što su Monte Carlo metode, koje mogu biti primjenjene samo kod razvijenih sistematičnih metoda numeričkog uzorkovanja osnovnih promjenljivih *X,* kao što je Markov lanac metoda simulacije prezentirana u radu [3].

U radu [1] opisane su dvije metode Monte Carlo simulacije: direktna metoda simulacije i uzorkovanje prema važnosti. Direktna metoda simulacije obuhvata uzorkovanje osnovnih nekoreliranih varijabli prema njihovim odgovarajućim probabilističkim karakteristikama i njihovo uvođenje u funkciju performansi *Z*, koja može biti izražena u funkciji od osnovnih slučajno promjenljivih X_i za odgovarajuća dejstva i čvrstoću konstrukcije

In practice the density function is not known and at the most some information is available only about a set of *n* basic random variables *X* which define the structural problem (e.g. mechanical and geometrical properties, dead and live loads, prestressing actions, etc.). Moreover, in concrete design the limit states are usually formulated in terms of functions of random variables *Y=Y(X)* which describe the structural response (e.g. stresses, strains, etc.), and such derivation is generally only available in an implicit form. Therefore a numerical approach is required. For solving random variable systems have been developed 3 methods of solution:

1. **Accurate method,** which requires as input known probability distribution functions for the variable. In the process of resolving apply numerical integration and Monte Carlo methods. This procedure requires extensive computer analysis.
2. **FOSM** (First order second moment method), where in the process of solving the use Taylor's series.
3. **PEM** (Point estimate method) that is most used in the analysis because it allows a gradual development of solutions.

Analytical integration is possible only in special cases. Numerical solution of the problem is simpler, but when it is integral to more than two random variables numerical integration can not in all cases provide sufficient accurate solution. With the increasing number of variables significantly increasing the calculation requirements, because in these cases integration areas are complex geometric figure, n-dimensional sphere. Therefore, solutions based on numerical integration for the specific methods were developed, such as Monte Carlo simulation, which can be applied only to developed systematic methods of numerical sampling of basic variables *X,* or Markov chain simulation method presented in the paper [3].

The paper [1] described two Monte Carlo Simulation Methods: the direct simulation method and importance sampling. The direct simulation method comprises drawing samples of the basic noncorelated variables according to their corresponding probabilistic characteristics and then feeding them into the performance function *Z*, that can be expressed in terms of basic random variables X_i for relevant loads and structural strength

$$Z = Z(X_1, X_2, X_3, ..., X_n) = R - S \tag{49}$$

Srednja vrijednost i varijanca vjerovatnoće nezadovoljavajućih performansi,

The mean and the variance of the unsatisfactory performance probability

$$P_u = \int ... \int f_{X_1 ... X_n}(x_1, ..., x_n) dx_1 ... dx_n \tag{50}$$

može se izraziti kao

$$\overline{P_u} = \frac{N_u}{N} \tag{51}$$

uz pretpostavku da je N_u broj ciklusa simulacije za *Z<0* u ukupnom broju ciklusa simulacije *N*. Kod manjih vjerovatnoća nezadovoljavajućih performansi potreban je veći broj ciklusa simulacije. Ovaj nedostatak može se prevazići korištenjem uzorkovanja prema važnosti.

U ovoj metodi osnovna slučajno promjenljiva generirana je u saglasnosti sa pažljivo odabranom raspodjelom vjerovatnoće (funkcija gustoće važnosti $h_{\underline{X}}(\underline{x})$) sa sred-

can be expressed as

$$Var\left(\overline{P_u}\right) = \frac{\left(1 - \overline{P_u}\right)\overline{P_u}}{N} \tag{52}$$

with assuming N_u to be the number of simulation cycles for which *Z<0* in a total *N* simulation cycles. By smaller unsatisfactory performance probabilities require larger numbers of simulation cycles. This deficiency can be overcome by using importance sampling.

In this method, the basic random variables are generated according to some carefully selected probability distributions (important density function $h_{\underline{X}}(\underline{x})$) with njim vrijednostima koje su bliže proračunskim vrijednostima nego njenoj originalnoj raspodjeli vjerovatnoće. Jednačina (50) za ovu metodu ima oblik

$$\overline{P_u} = \frac{1}{N}\sum_{i=1}^{N} I_i \frac{f_{\underline{X}}(x_{1i},..., x_{ni})}{h_{\underline{X}}(x_{1i},..., x_{ni})} \tag{53}$$

gdje je

I – funkcija indikatora performansi sa vrijednostima 0 ili 1

$f_{\underline{X}}$ – originalna funkcija gustoće osnovnih slučajno promjenljivih

$h_{\underline{X}}$ – odabrana funkcija gustoće osnovnih slučajno promjenljivih

Primjena ove metode prezentirana je u radu [11].

Primjena Monte Carlo simulacije za analizu pouzdanosti armiranog betona i prednapregnutog betona data je u radu [4], na primjeru lučnog mosta.

U radu [5] opisana je primjena Fuzzy teorije u odnosu na teoriju vjerovatnoće. Fuzzy teorija omogućava analizu nesigurnosti u slučaju nedostatka informacija za razliku od teorije vjerovatnoće, koja je zasnovana na potpunom znanju o stohastičkim promjenama koje su rezultat slučajne prirode određenih veličina. Korištenjem Fuzzy kriterija sve nesigurnosti mogu se modelirati kroz interval vrijednosti, ograničen odgovarajućim minimalnim i maksimalnim ekstremom. Fuzzy funcija pripadnosti $\mu = \mu_{\tilde{A}}(x)$ fuzzy seta $\tilde{A} \subseteq X$, za određeni nivo pripadnosti $\alpha \in [0,1]$, je moguća raspodjela pogodna za opis informacija o nesigurnosti, tamo gdje raspodjela vjerovatnoće nije direktno dostupna.

mean values that are closer to the design value than their original probability distributions. Equation (50) for this method have form

where

I – performance indicator function that takes value of 0 or 1

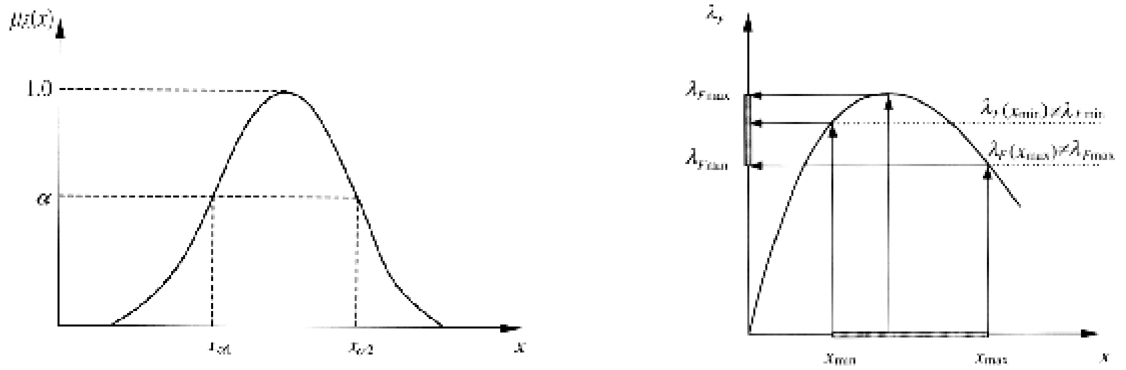
$f_{\underline{X}}$ – the original density function of the basic random variables

$h_{\underline{X}}$ – the selected density function of the basic random variables

Application of these methods is presented in the paper [11].

Application of Monte Carlo simulation for reliability analysis of reinforced concrete and prestressed concrete is provided in the paper [4], for example arc bridge.

The paper [5] describes the application of fuzzy theory in relation to the probability theory. Fuzzy theory allows analysis of uncertainty in case of lack of information as opposed to the probability theory, which is based on a perfect knowledge about the stochastic variability resulting from the random nature of the same quantities. Using a fuzzy criterion all uncertainties can be modeled through bands of values, bounded between suitable minimum and maximum extremes. A fuzzy membership function $\mu = \mu_{\tilde{A}}(x)$ of a fuzzy set $\tilde{A} \subseteq X$, for defined level of membership $\alpha \in [0,1]$, is a possibilistic distribution suitable to describe uncertain information, when a probabilistic distribution is not directly available.



Sl.14. (a)Pripadajuća funkcija i α nivo; (b)Mapiranje između intervala nepouzdanosti na osi x i odgovarajućeg intervala odgovora na osi λ_F [5]

Fig.14. (a)Membership function and α level; (b)Mapping between the interval of uncertainty on x and the corresponding response interval on λ_F [5]

Primjena Taylor-ovih redova zahtjeva formulisanje i rješavanje derivacija, što je za viševarijabilne probleme zahtjevan zadatak, posebno kada je funkcija zadana implicitno u vidu krive, grafa ili kao rješenje MKE. Pojednostavljeni model pouzdanosti, prvobitno predstavljen kod projektovanja čeličnih konstrukcija, koristi samo srednju vrijednost i koeficijent varijacije za otpornost *R* i dejstva *S* u pojedinim graničnim stanjima da bi se dobio indeks pouzdanosti β, koji se proračunavao kao,

Application of Taylor's series requests in formulating and solving the derivative, which is for multi-variable problems demanding task, especially when the function is implicit in the form of the default curve, a graph or as a FEM solution. A simplified reliability model, first introduced in structural steel design, only uses the mean values and coefficients of variation for the resistance R and load S in a particular limiting state to obtain the reliability index β, which is computed as

$$\beta = \frac{\ln(\mu_R / \mu_S)}{\sqrt{V_R{}^2 + V_S^2}} \tag{54}$$

nezavisno od tipa raspodjele *R* i *S*.

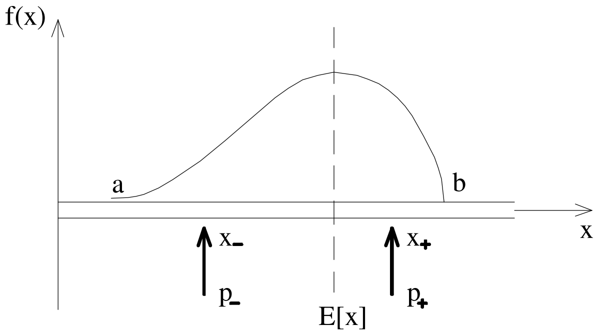
Primjeri su prezentirani u radovima [7], [14] i [16].

PEM metodu je prezentirao Rosenblueth 1975 g., a potpuno je razvijena 1981 g. Metoda je zasnovana na analogiji između distribucije vjerovatnoće i distribuiranog vertikalnog opterećenja na horizontalnoj krutoj gredi. Očekivana vrijednost odgovara položaju djelovanja sile koja uravnotežuje opterećenje (jedinična vrijednost) ili težištu opterećenja. Rosenblueth je predložio da se parametri očekivane vrijednosti i standardne devijacije odrede iz analogije sa slobodno oslonjenom gredom.

regardless of the type of distribution of R and S.

Examples are presented in the papers [7], [14] and [16].

PEM method was presented by Rosenblueth 1975, and is fully developed in 1981. Method is based on the analogy between probability distributions and distributed vertical load on the horizontal solid beam. Expected value corresponds to the position of action that balances the load force (unit value) or the center of gravity of the load. Rosenblueth proposed that the parameters of the expected value and standard deviation determine according to the analogy with the simple beam.



Sl.15 – Distribucija vjerovatnoće – analaogija sa krutom gredom
Fig.15 – Distribution of probability – solid beam analogy

Reakcije p- i p+ su dvije ravnotežne tačke funkcije distribucije f(x). Primjenom postavki teorije pouzdanosti mogu se postaviti slijedeće jednačine:

- Uslov ravnoteže:

Reactions *p-* and *p +* are two points of equilibrium of distribution function *f(x)*. Applying the theory of reliability can set the following equation:

- The condition of equilibrium:

$$p_+ + p_- = 1 \tag{55}$$

- Očekivana vrijednost:
- Expected value:

$$p_+x_+ + p_-x_- = E[f(x)] = \bar{x} \tag{56}$$

- Mjera rasipanja
- The measure of dispersion

$$p_+(x_+ - \bar{x})^2 + p_-(x_- - \bar{x})^2 = \sigma[f(x)]^2 = \sigma^2[x] \tag{57}$$

- Asimetrija raspodjele:
- Asymmetry of distribution:

$$p_+(x_+ - \bar{x})^3 + p_-(x_- - \bar{x})^3 = \beta(1)\sigma^3[x] \tag{58}$$

Rješenje jednačina (55) do (58) su:

Solutions of equations (55) to (58) are:

$$p_+ = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{1}{1 + \left[\frac{\beta(1)}{2}\right]^2}}\right];\; p_- = 1 - p_+;\; x_+ = \bar{x} + \sigma[x]\sqrt{\frac{p_-}{p_+}};\; x_- = \bar{x} - \sigma[x]\sqrt{\frac{p_+}{p_-}} \tag{59}$$

Kod standardizirane normalne raspodjele može se usvojiti $\beta(1) = 0$ tako da izrazi (59) imaju oblik:

With standardized normal distribution can be adopted $\beta(1) = 0$ so that expressions (59) have the form:

$$p_+ = p_- = \frac{1}{2};\; x_+ = \bar{x} + \sigma[x];\; x_- = \bar{x} - \sigma[x] \tag{60}$$

Na osnovu određenih tačaka slučajno promjenjivih *x* i funkcionalne zavisnosti *x* i *y* dobiju se vrijednosti funkcije *y(x), y +* i *y-*, korištenjem izraza,

On the basis of certain points of random variable *x* and the functional dependence *x* and *y* get the value of the function *y(x), y +* and *y-*, using the expression,

$$E\left[y^{M}\right]= p_{-}y_{-}^{M} + p_{+}y_{+}^{M} \qquad (61)$$

gdje *M* odgovara broju poznatih momenata za slučajno promjenjivu *x*.

where *M* corresponds to the number of known moments for random variable *x*.

5 ANALIZA POUZDANOSTI VEZE PREFABRIKOVANE PLOČE I MONOLITNOG ZIDA

U periodu 2004. – 2008. godina provedena su eksperimentalna i numerička istraživanja veze prefabrikovane ploče i monolitnog zida (slika 16), sa ciljem definisanja mehanizma rada ovakve veze. Detalji istraživanja i rezultati istraživanja prezentirani su u radovima [23] i [24].

Numerički modeli su urađeni korištenjem MKE i Link elemenata modeliranih korištenjem eksperimentom dobijenog *M*-ϕ radnog dijagrama (slika 17) [24]. Uporedni rezultati provedenih numeričkih istraživanja prezentirani su na slici 18. Istraživanjem je utvrđeno da i monolitne veze imaju određen stepen popustljivosti. Iz mnoštva rezultata ovdje se daju dvije vrijednosti stepena popustljivosti prefabrikovane veze u odnosu na monolitnu:

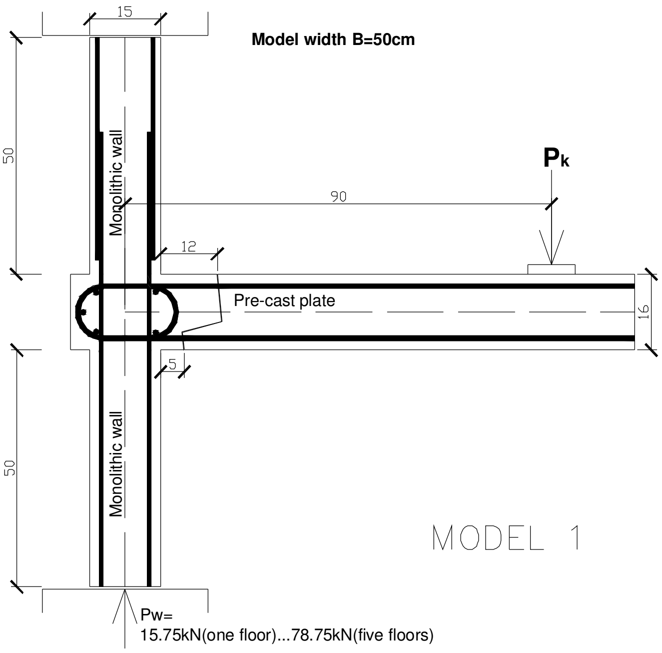
- Eksploataciono opterećenje - $\gamma_{calc.,serv.} = 0.92$
- Granično opterećenje sa faktorom sigurnosti 1.75 - $\gamma_{calc.,1,75} = 0.90$

5 RELIABILITY OF THE PRECAST PLATE-MONOLITHIC WALL CONNECTION

In the period 2004^{th}-2008^{th} years, were performed experimental and numerical research of precast plate – monolithic wall connection (fig.16), with the aim of defining the work mechanism of such connections. Details of the research and results are presented in the papers [23] and [24].

Numerical models are done using FEM and Link elements modeled in the experiment obtained using M-ϕ working diagram (Fig.17) [24]. Comparative results of the conducted numerical studies are presented in Figure 18. Research has found that the monolithic connection have a certain degree of yielding. From the many results here are two values of the degree of prefabricated connections yielding relatively to the monolithic:

- Serviceability load - $\gamma_{calc.,serv.} = 0.92$
- Ultimate load with safety factor 1.75 - $\gamma_{calc.,1,75} = 0.90$



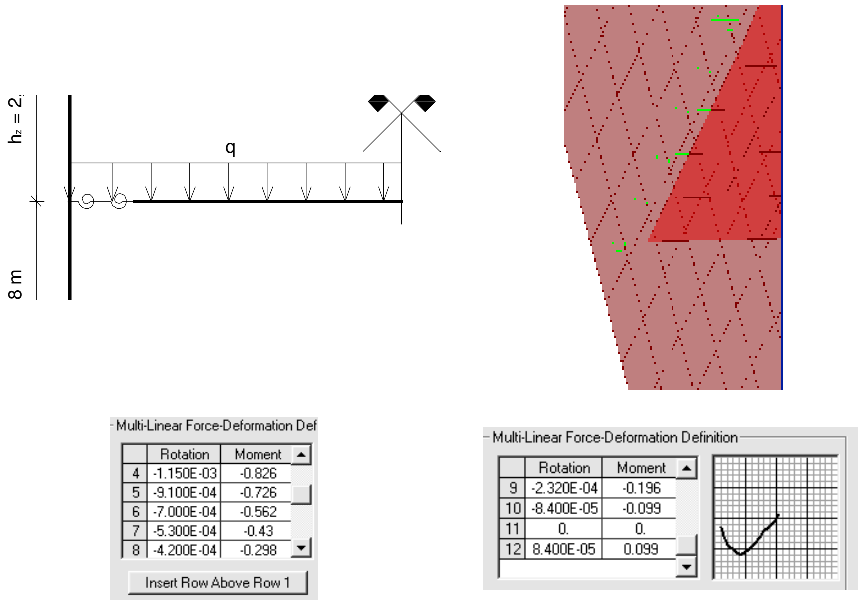
Sl. 16 – Montažna veza montažne ploče i monolitnog zida
Fig. 16 – Precast connections of precast plate and monolithic wall

U radu [23] detaljno su dati rezultati eksperimentalnog istraživanja. Ovdje se daju uporedne vrijednosti stepena popustljivosti prefabrikovane veze u odnosu na monolitne:

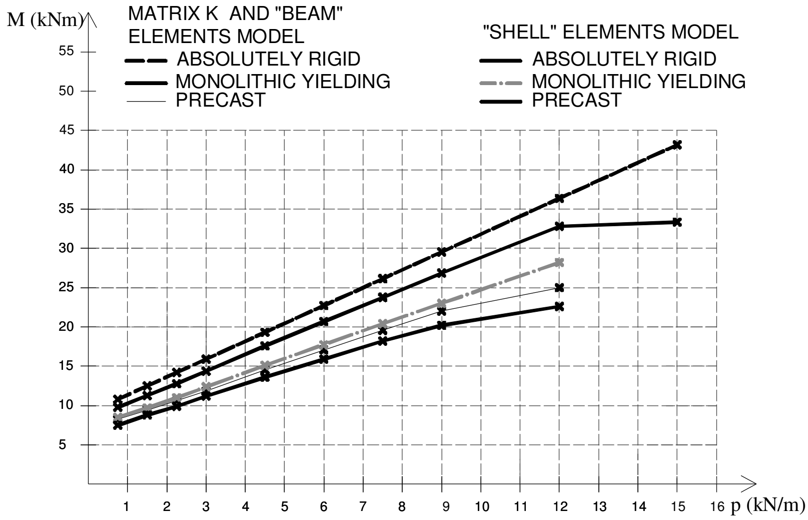
- $\gamma_{exp,serv.} = 0.907 \div 0.913$
- $\gamma_{exp,1,75} = 0.897 \div 0.903$

The paper [23] have detailed the results of experimental research. Here give comparative values of the degree of prefabricated connections yielding relatively to the monolithic:

- $\gamma_{exp,serv.} = 0.907 \div 0.913$
- $\gamma_{exp,1,75} = 0.897 \div 0.903$



Sl. 17 – MKE model [*24*]
Fig. 17 – FEM Model [*24*]



Sl.18. Računske krive oslonački moment - opterećenje (M-p) za analizirane numeričke modele
Fig.18. Calculation curves support moment – load (M-p) for analyzed numerical models

Uporedba rezultata pokazuje zadovoljavajuću tačnost numeričkih modela. Da bi se verificirali prezentirani rezultati neophodna je analiza pouzdanosti dobijenih karakterističnih vrijednosti u cilju definisanja stepena sigurnosti (faktora sigurnosti) analiziranog prefabrikovanog sistema građenja.

5.1 Analiza popustljivosti veze

Provedena analiza pouzdanosti stepena popustljivosti istraživane veze prefabrikovane ploče i monolitnog zida definisana je izrazom datim u radu [8],

Comparison of results showed a satisfactory accuracy of the numerical model. To verify the presented results is necessary reliability analysis of the obtained characteristic values with the aim of defining the safety degree (factor) of analyzed prefabricated building systems.

5.1 Analysis of the connection yielding

Reliability analysis of yielding degree of researched precast plate-monolithic wall connection, defined by expression given in the paper [8], was performed

$$\alpha = \frac{S_{\phi} \cdot L_{pl.}}{B_{pl.}} = \frac{S_{\phi} \cdot L_{pl.}}{E \cdot I_{pl.}} \qquad (62)$$

Analiza je urađena uz pretpostavku normalne funkcije raspodjele vjerovatnoće za uticajne parametre. Raspon ploče je usvojen konstantan L_{pl} *=6,15m*, a koeficijent iskrivljenosti raspodjele $\beta_1=0$. Dakle, analiziran je stepen popustljivosti kao funkcija tri slučajno promjenljive $\alpha = f(S_\phi, E, I)$.

Pošto je ispitivanje provedeno na 3 monolitna i 3 montažna modela ne postoji dovoljan broj podataka za statističku obradu podataka, odnosno pouzdanu determinaciju parametara rasipanja rezultata. Stoga su obrađeni rezultati ispitivanja upoređivani sa do sada provedenim istraživanjima i preporukama prezentiranim u [6], [9], [14], [18] i [19].

Tako je Ellingwood 1980.godine, na osnovu provedenih opsežnih eksperimenata, preporučio reprezentativne vrijednosti koeficijenta varijacije, i to za betonske elemente izložene savijanju 8÷9,5%. Isti autor je definisao za spoj greda-stub koeficijent varijacije 10%. Mehlhorn je u [18] prezentirao opsežna istraživanja smicanja glatkih spojnica gdje je za 176 opita dobijen koeficijent varijacije 16%. Iste rezultate su dobili Hansen, Olesen, Fauchart i Cortini. Pommeret je dobio koeficijent varijacije 12÷15%, Laing 17÷18%, Pume 13%. U radu [13] je prezentirana statistička obrada rezultata ispitanih 95 modela za koje je dobijen koeficijent varijacije 12%.

Na osnovu prethodnih navedenih podataka i rezultata eksperimenata analiza je provedena za koeficijente varijacije *10 - 25 %.* Za očekivanu vrijednost je usvojena eksperimentima dobijena srednja vrijednost $S_\phi = 3507{,}94\ kNm/rad$.

Kod proizvodnje prefabrikovanih elemenata veličina dopuštene greške zavisi od tipa konstrukcije, načina proizvodnje i montaže elemenata i najčešće se kreće u granicama od 3 ÷10 mm **Error! Reference source not found.**.

Za potrebe definisanja pouzdanosti u ovom radu usvojena je tolerancija za širinu ploče modela ±10mm, tj. $b_{pl.} = 500 \pm 10\ mm$, a za debljinu ploče ±10mm, tj. $d_{pl.} = 160 \pm 3\ mm$.

Uz pretpostavku standardizovane normalne raspodjele i karakterističnih 5%-fraktilnih vrijednosti dozvoljenih odstupanja, koeficijenti varijacije su:

- za širinu modela CV = 1,22 % (usvojeno 1,5%)
- za debljinu ploče CV = 1,13 % (usvojeno 1,5%)

Razlika eksperimentom utvrđenih reprezentativnih vrijednosti modula elastičnosti betona monolitnih i montažnih modela je 4,78%. Ova razlika ima odraz na različite deformacione karakteristike. Međutim provedena eksperimentalna i numerička istraživanja su pokazala da utvrđena veličina razlike nije imala značajan uticaj na uporednu analizu stepena popustljivosti monolitnih montažnih modela. Pošto su izrazi za određivanje modula elastičnosti u funkciji čvrstoće na pritisak betona, ovdje se u nedostatku dovoljnog broja podataka uzimaju preporučene vrijednosti koeficijenata varijacije, zasnovane na opsežnim istraživanjima pojedinih autora.

Ellingwood je 1980. god. predložio za armirani betonski element opterećen na savijanje koeficijent varijacije 14% [14]. Bljuger je prilikom analize spojeva zidova usvojio koeficijent varijacije *CV=12,8%* [6]. Zasnovano na rezultatima istraživanja koja su uradili Entroy 1960, Murdock 1953, Rusch 1969, Mirza 1979,

The analysis is done assuming a normal probability density function for the influential parameters. The span of plate was adopted L_{pl} =6,15m, and coefficient of distribution distortion $\beta_1=0$. Therefore, the yielding degree as a function of three random variables $\alpha = f(S_\phi, E, I)$ was analyzed.

Because the testing performed on the 3 monolithic and 3 precast models 3 was not enough data for statistical data processing, and reliable determination of scattering results parameters. Therefore, the obtained results of experimental research compared with results of so far performed research and recommendations presented in [6], [9], [14], [18] and [19].

On the basis of extensive experiments Ellingwood 1980's recommended a representative coefficient of variations, for concrete elements exposed to bending, from 8 to 9.5%. The same author has defined for the beam-column connection coefficient of variation of 10%. Mehlhorn in [18] presented extensive studies of smooth shear connectors, where for the 176 experiments obtained coefficient of variation 16%. The same results were obtained Hansen, Olesen, Fauchart and Cortini. Pommeret got a coefficient of variation 12-15%, Laing 17 ÷18%, Pume 13%. The paper [13] presented a statistical analysis of the testing results with 95 models, for which the obtained coefficient of variation of 12%.

Based on these previous data and results of experiments, analysis for the coefficient of variation from 10 to 25% was performed. The experiment obtained mean value $S_\phi = 3507{,}94\ kNm/rad$ was adopted as the expected value.

By production of precast elements, size of permitted deviation depends on the type of structures, production procedure and erection procedure, and often is within the limits of 3 to 10mm **Error! Reference source not found.**.

For the purposes of the definition of reliability in this paper, tolerance for the width of panel models was adopted ±10mm, ie., $b_{pl.} = 500 \pm 10\ mm$, and for a plate thickness ±10mm, ie., $d_{pl.} = 160 \pm 3\ mm$.

Assuming standardized normal distribution and typical 5%-fractil values of permitted deviations, coefficients of variation are:

- for width of the model CV = 1.22 % (adopted 1.5%)
- for plate thickness CV = 1.13% (adopted 1.5%)

Difference of value of modulus of elasticity of precast and monolithic model concrete determined with experiment is 4.78%. This difference is reflected in different deformation characteristics. However, conducted experimental and numerical studies have shown that defined size of differences had no significant effect on the comparative analysis of the yielding degree of monolithic and precast models. Since the expressions for determining the modulus of elasticity in the function of compressive strength of concrete, here is the lack of sufficient data to take the recommended values of coefficients of variations, based on extensive research of individual authors.

Ellingwood in 1980. proposed for reinforced concrete elements loaded in bending coefficient of variation of 14% [14]. Bljuger during the analysis of compound walls adopted coefficient of variation CV = 12.8% [6]. Based on the results of research that is done Entroy 1960, Melchers je prezentirao u tabeli 8.7 [19] koeficijente varijacije, odnosno standardne devijacije koje se kreću od 2,8 MPa za odličan beton do 5,6 MPa za loš beton. Ovo su rezultati za čvrstoće na pritisak *fc* > 28 MPa. Za čvrstoću na pritisak nominalne vrijednosti 30 MPa koeficijent varijacije je: *CV*=9,33% za odličan beton, a *CV*=18,66% za loš beton.

Za analizu pouzdanosti u ovom radu usvojeni su koeficijenti varijacije kao i kod krutosti spoja 10 - 25 %. Usvojena očekivana vrijednost je $E_c = 34975\ MN/m^2$.

Za funkciju tri slučajno promjenljive $y = y\,(x_1, x_2, x_3)$ vrijedi:

Murdock 1953, Rusch 1969, Mirza 1979, Melchers is presented in Table 8.7 [19] coefficient of variation or standard deviation ranging from 2.8 MPa for the excellent concrete to 5.6 MPa for the bad concrete. These are the results of compressive strength *fc* > 28 MPa. For compressive strength with a nominal value 30 MPa the coefficient of variation is *CV* = 9.33% for the excellent concrete, and *CV* = 18.66% for the bad concrete.

For reliability analysis in this paper have been adopted the coefficient of variation as for connection stiffness from 10 to 25%. Expected value is adopted experimental mean value $E_c = 34975\ MN/m^2$.

For the function of three random variable $y = y\,(x_1, x_2, x_3)$ is valid:

$$y_{\pm\pm\pm} = y(\overline{x_1} \pm \sigma[x_1], \overline{x_2} \pm \sigma[x_2], \overline{x_3} \pm \sigma[x_3]) \tag{63}$$

Težinski koeficijenti *p* su:

Weight coefficients *p* are:

$$p_{+++} = p_{---} = \frac{1}{2^3}(1+\rho_{12}+\rho_{23}+\rho_{31});\ p_{++-} = p_{--+} = \frac{1}{2^3}(1+\rho_{12}-\rho_{23}-\rho_{31})$$

$$p_{+-+} = p_{-+-} = \frac{1}{2^3}(1-\rho_{12}-\rho_{23}+\rho_{31});\ p_{+--} = p_{-++} = \frac{1}{2^3}(1-\rho_{12}+\rho_{23}-\rho_{31}) \tag{64}$$

Očekivana vrijednost je:

Expected value is:

$$E[y^M] = p_{+++}y^M_{+++} + p_{++-}y^M_{++-} + \ldots + p_{---}y^M_{---} \tag{65}$$

Pošto je moment inercije ***I*** funkcija dvije slučajno projenljive $I = f(b_{pl}, h_{pl})$, prethodno je urađena analiza statističkih parametara momenta inercije. Statističkom obradom podataka za širinu modela i debljinu ploče dobijen je koeficijent korelacije: $\rho_{bpl.hpl.} \approx 0$. Prema tome težinski koeficijenti su:

Since the moment of inertia *I* is function of two random variable $I = f(b_{pl}, h_{pl})$, analysis of the preliminary statistical parameters of the moment of inertia were performed. Correlation coefficient of model width and thickness of plate was adopted $\rho_{bpl.hpl.} \approx 0$. Accordingly, weight coefficients are:

$$p_{++} = p_{+-} = p_{-+} = p_{--} = \frac{1}{4};\ I = \frac{b_{pl.} \cdot h^3_{pl}}{12}$$

a proračunata varijacija,

and calculated variation is

$$V(I) = 5{,}97\ \%$$

U daljnjem proračunu usvojena je varijacija *V(I)* = 6,0%.

U radu je prezentirana analiza jednog statističkog slučaja. Pregled svih analiziranih statističkih slučajeva daje se u tabeli 1.

In further calculations was adopted *V(I)* = 6,0%.

The paper presented the statistical analysis of one case. Review of the all statistical cases is given in Table 1.

Slučaj (a)

Case (a)

$$\bar{I} = 0{,}00003392\ m^4 \qquad V(I) = 6\ \%$$

$$\bar{E}_c = 34975\ MN/m^2 \qquad V(E_c) = 10\ \%$$

$$\bar{S}_\phi = 3507{,}94\ kNm/rad \qquad V(S_\phi) = 6\ \%$$

L=6,15 m=const.

Koeficijenti korelacije eksperimentalnih rezultata su: $\rho_{I,E_C} = 0{,}4; \rho_{I,S_\phi} = 0{,}9; \rho_{E,S_\phi} = 0{,}6$

Correlation coefficients of experimental results are: $\rho_{I,E_C} = 0{,}4; \rho_{I,S_\phi} = 0{,}9; \rho_{E,S_\phi} = 0{,}6$

$$I_+ = 0{,}000035955 \qquad E_{C+} = 38472{,}5 \qquad S_{\phi+} = 3858{,}73$$

$$I_- = 0{,}000031884 \qquad E_{C-} = 31477{,}5 \qquad S_{\phi-} = 3157{,}15$$

Težinski koeficijenti su:

Weighting factors are:

$$p_{+++} = p_{---} = \frac{1}{8}(1+0{,}4+0{,}9+0{,}6) = 0{,}3625$$

$$p_{++-} = p_{--+} = \frac{1}{8}(1+0,4-0,6-0,9) = -0,0125$$

$$p_{+-+} = p_{-+-} = \frac{1}{8}(1-0,4-0,6+0,9) = 0,1125$$

$$p_{+--} = p_{-++} = \frac{1}{8}(1-0,4+0,6-0,9) = 0,0375$$

$\alpha(I, E_C, S_\phi)$	α_{ijk}	p_{ijk}	$\alpha_{ijk} \cdot p_{ijk}$	α_{ijk}^2	$\alpha_{ijk}^2 \cdot p_{ijk}$
α_{+++}	17,036	0,3625	6,17555	290,225	105,2066
α_{++-}	13,939	−0,0125	−0,17424	194,296	−2,4287
α_{+-+}	20,968	0,1125	2,3589	439,657	49,4614
α_{+--}	17,156	0,0375	0,64335	294,328	11,0373
α_{-++}	19,346	0,0375	0,725475	374,268	14,0351
α_{-+-}	15,829	0,1125	1,78076	250,557	28,1877
α_{--+}	23,645	−0,0125	−0,29556	559,086	−6,9886
α_{---}	19,346	0,3625	7,01293	374,268	135,6721
			18,227		334,183

$$E[\alpha] = 18,227; \quad E[\alpha^2] = 334,183$$

$$V[\alpha] = 334,183 - (18,227)^2 = 1,95947$$

$$\sigma[\alpha] = 1,3998$$

Varijacija stepena popustljivosti je:

Variation of yielding degree is:

$$V(\alpha) = \frac{1,3998}{18,227} \cdot 100 = 7,68\ \%$$

Na osnovu provedene statističke analize određena je minimalna vrijednost stepena popustljivosti za koju je vjerovatnoća manjih vrijednosti 5 % (karakteristična 5%-fraktilna vrijednost). Proračun je proveden primjenom standardizovane normalne raspodjele.

On the basis of the statistical analysis determined the minimum value of the degree of yielding, for which is probability of smaller value of 5% (typical 5% fractil value). The calculation is performed using the standardized normal distribution.

$$\alpha_{0,05} = 18,227 - 1,64 \cdot 1,3998 = 15,931$$

Za proračunatu vrijednost oslonački moment će biti:

For above calculated value moment on support will be:

$$M_{osl.} = 0,888 \cdot \overline{M}$$

Tabela 1 – Analizirani statistički slučajevi
Table 1 – Analyzed statistical cases

Statictical cases	Moment of inertia I (m^4)		Modulus of elasticity E_c (MN/m^2)		Connection stiffness S_ϕ (kNm/rad)		Degree of yielding γ[1]	
	E[I]	V(I)%	E[E_c]	V(E_c)%	E[S_ϕ]	V(S_ϕ)%	V(γ)%	$\gamma_{0,05}$
Case (a)	0,00003392	6,0	34975	10,0	3507,94	10,0	7,68	0,888
Case (b)	0,00003392	6,0	34975	15,0	3507,94	15,0	11,31	0,878
Case (c)	0,00003392	6,0	34975	20,0	3507,94	25,0	26,17	0,839
Case (d)	0,00003392	6,0	34975	25,0	3507,94	25,0	20,61	0,860
Case (e)	0,00003392	6,0	34975	10,0	3507,94	25,0	15,90	0,869
Case (f)	0,00003392	6,0	34975	25,0	3507,94	10,0	21,91	0,860

[1] $\gamma = \frac{\alpha}{2+\alpha}$

Na osnovu provedene analize usvojena je maksimalna očekivana varijacija 27 %, odnosno zaokružena najveća vrijednost iz tabele 1.

Karakteristična 5 % fraktilna vrijednost α je,

Based on the performed analysis adopted the maximum expected variation of 27%, ie., maximum value from Table 1.

Characteristics 5%-fractil value of α is,

$$\alpha_{0,05} = 18,276 - 1,64 \cdot 4,2935 = 10,183$$

odnosno moment nad osloncem,

or the moment on support is,

$$E(M_{osl.,0.05}) = 0,835 \cdot \overline{M}$$

Varijacija momenta nad osloncem je: $V(M_{osl.,0.05}) = 7,22\%$.

Variation of moment on support is: $V(M_{osl.,0.05}) = 7,22\%$.

5.2 Analiza vjerovatnoće otkaza veze

Ellingwood je 1980. god., na osnovu statističke obrade rezultata mjerenja na objektima, dao prijedlog reprezentativnih vrijednosti varijacija za stalno i pokretno opterećenje i to:

- za stalno opterećenje *V(g)* = 10%
- za pokretno opterećenje *V(p)* = 25%

Pošto je kod opterećenja ispitanih spojeva odnos *g:p* = 2:1, varijacija ukupnog opterećenja je *V(q)* = 15%. Očekivana vrijednost momenta nad osloncem (u spoju), za eksploataciono opterećenje, je $E(M_q)$=14,17 kNm.

Uz usvojene varijacije za stalno i pokretno opterećenje, granične vrijednosti uticajnih momenata su $M_{q,min}$=12,04 kNm i $M_{q,max}$=16,30 kNm.

Srednja vrijednost momenta nosivosti utvrđena eksperimentom je: $\overline{M}_n = 42,67\ kNm$

Uz uvažavanje analize pouzdanosti stepena popustljivosti spoja, provedene u prethodnoj tački, minimalna i maksimalna vrijednost momenta nosivosti spoja je $M_{n,min}$=39,58 kNm i $M_{n,max}$=45,75 kNm.

Mjerodavni statistički parametri su:

$$\mu_z = \overline{M}_n - \overline{M}_q = 42,67 - 14,17 = 28,5\ kNm$$

$$\sigma_z^2 = \sigma_{Mn}^2 + \sigma_{Mq}^2$$

$$\sigma_{Mn}^2 = (0,0722 \cdot 42,67)^2 = 9,491\ (kNm)^2$$

Indeks pouzdanosti je,

5.2 Analysis of the probability of connection failure

On the basis of statistical processing results of measurements on objects, Ellingwood 1980th proposed representative values of variation for dead and live loads as follows:

- for dead load *V(g)* = 10 %
- for live load *V(p)* = 25 %

Since the load ratio of researched connection *g:p* = 2:1, the variation of the total load is *V(q)* = 15%. Expected value of moment on support (in connection area), for service load, is $E(M_q)$=14,17 kNm.

With variations adopted for the dead and live loads, the limit value of moments are $M_{q,min}$=12,04 kNm and $M_{q,max}$=16,30 kNm.

Average value of moment capacity determined in the experiment is: $\overline{M}_n = 42,67\ kNm$

With the appreciation of the reliability analysis of degree of yielding, implemented in the preceding paragraph, the minimum and maximum value of connection moment capacity is $M_{n,min}$=39,58 kNm and $M_{n,max}$=45,75 kNm.

The relevant statistical parameters are:

$$\sigma_{Mq}^2 = (0,15 \cdot 14,17)^2 = 4,518\ (kNm)^2$$

$$\sigma_z^2 = 9,491 + 4,518 = 14,009$$

Reliability index is,

$$\beta = \frac{28,5}{3,743} = 7,61$$

odnosno vjerovatnoća otkaza je,

or probability of failure is,

$$P_f = \phi(-7,61) = 1,445 \cdot 10^{-14}\ .$$

Eksperimentom je utvrđena varijacija momenta loma 10 %. U tom slučaju je indeks pouzdanosti,

Experimental value of variaton of failure moment is 10%. In this case, the reliability index is,

$$\beta = \frac{28,5}{4,767} = 5,98$$

a vjerovatnoća otkaza,

and the probability of failure,

$$P_f = \phi(-5,98) = 1,325 \cdot 10^{-9}\ .$$

U tabeli 2 date su preporučene minimalne vrijednosti indeksa pouzdanosti za klase pouzdanosti, preuzete iz reference[10].

Indeks pouzdanosti je definisan za početnu krutost prefabrikovane veze. Ako je indeks pouzdanosti $\beta < 2$, u radu [15] preporučuje se uvođenje u analizu rezidualne krutosti veze.

In table 2 are given recommended minimum values of reliability index for reliability classes RC, taken from reference [10].

Reliability index is determined by the initial stiffness of the precast connection. If the reliability index $\beta < 2$, the paper [15] recommends the introduction to the analysis of residual connection stiffness.

Tabela 2 – Preporučene minimalne vrijednost indeksa pouzdanosti β (ULS) [10]
Table 2 – Recommended minimum values of reliability index β (ULS) [10]

Reliability classes	Minimum values for β	
	Reference period of 1 year	Reference period of 50 years
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

6 ZAKLJUČCI

Analiza građevinskih konstrukcija podrazumjeva viševarijabilne probleme, gdje su skoro sve varijable stohastičke. Prvi korak u analizi pouzdanosti je definisanje funkcije gustoće vjerovatnoće pojedinih varijabli. Postupci koji se primjenjuju svode se na upoređivanje zakonitosti promjene varijable sa uobičajenim zakonitostima koje se koriste u građevinarstvu, opisanim u radu. Pouzdanost definisanja zakonitosti je veća što je veća baza podataka provedenih istraživanja određene varijable. Kod prefabrikovane gradnje pouzdanost prije svega zavisi od pouzdanosti veza prefabrikovanih elemenata. Realan mehanizam rada veze može se utvrditi samo eksperimentom. Stoga kod prefabrikovanih veza ostaje problem unificiranja postupaka analize. Povećanje baze podataka o ponašanju pojedinih montažnih veza daje mogućnost za primjenu analize pouzdanosti u ovom području građevinarstva. U tom smislu analiza pouzdanosti prezentirana u ovom radu povećava postojeću bazu statističkih podataka potrebnih za definisanje pouzdanosti prefabrikovanih veza. Analiza je urađena za eksperimentom utvrđene srednje vrijednosti i varijacije, kao i za varijacije preporučene od autora koji su se bavili razmatranom problematikom. Za analizu je usvojena normalna raspodjela slučajno promjenljivih. Koeficijenti korelacije između pojedinih slučajno promjenjivih su usvojeni za uslove eksperimenta. Zbog nedovoljnog broja ispitanih uzoraka za primjenu pouzdane statističke procedure, koeficijenti korelacije su reducirani na očekivane minimalne vrijednosti. Usvojena je redukcija koeficijenta korelacije ρ_{I,S_ϕ} *i* ρ_{E_C,S_ϕ} za 20%. Naime, smatra se da slučajno promjenljive imaju jaku korelaciju ako je njihov koeficijent korelacije $\rho \geq 0{,}50$. Pošto je korelacija momenta inercije i krutosti spoja, kao i krutosti spoja i modula elastičnosti betona, kod ispitanih montažnih veza jaka, u sklopu diskusije su razmatrani koeficijenti korelacije umanjeni na vrijednost koja je na donjoj granici dobre korelacije.

Prema tome usvojeni statistički parametri su: $\rho_{I,E_C} = 0{,}4;\ \rho_{I,S_\phi} = 0{,}72;\ \rho_{E_C,S_\phi} = 0{,}48$.

Varijacija stepena popustljivosti i moment nad osloncem su:

6 CONCLUSIONS

Analysis of civil structures includes multivariable problems, where almost all stochastic variables. The first step in analyzing the reliability is definition of probability density function of individual variables. The procedures applied are reduced to comparing the law of the variable changes with the usual laws that are used in civil engineering, as described in the paper. Reliability of definition of laws is greater the larger the database conducted research specific variables. By prefabricated buildings the reliability mainly depends on the reliability of the precat connections. Realistic work mechanism of the connection can be established only in the experiment. Therefore, the precast connections remains problem of analysis procedures unification. Increase the database on the behavior of individual precast connections gives the possibility for the application of reliability analysis in this area of civil engineering. In this sense, the reliability analysis presented in this paper increases the existing base of statistical data required for defining the reliability of precast connections. Analysis is done for experiment determined the mean and variation, as well as variations recommended by authors who have dealt with issues under consideration. For the analysis adopted a normal distribution of random variable. Correlation coefficients between individual random variable adopted for the experiment conditions. Due to an insufficient number of tested samples for implementation of reliable statistical procedure, the correlation coefficients were reduced to the expected minimum values. Adopted is to reduce correlation coefficients ρ_{I,S_ϕ} and ρ_{E_C,S_ϕ} of 20%. It is considered that random variables have a strong correlation since the correlation coefficient is $\rho \geq 0{,}50$. Since the correlation between moment of inertia and connection stiffness, and connection stiffness and modulus of elasticity of concrete strong for researched precast connection, in the discussions were considered correlation coefficients reduced to a value that is at the lower limit of good correlation.

Accordingly, the correlation coefficients were adopted: $\rho_{I,E_C} = 0{,}4;\ \rho_{I,S_\phi} = 0{,}72;\ \rho_{E_C,S_\phi} = 0{,}48$

Variation of degree of yielding and moment on support are:

$$V(\alpha) = 29{,}03\,\%$$

$$V(M_{osl.}) = 8\,\% < V(M_{osl.,\exp.}) = 10\,\%$$

Indeks pouzdanosti ispitane prefabrikovane veze je β = 5,98.

Varijacija momenta nosivosti, za očekivane

Reliability index of researched precast connection is β = 5.98.

Variation of moment capacity, for the expected minimalne vrijednosti koeficijenata korelacije, je manja od usvojene za uslove eksperimenta, Na osnovu prethodne analize preporučuje se za potrebe definisanja pouzdanosti montažnih veza, koje se ponašaju slično kao ispitane predmetne montažne veze, koristiti sljedeće minimalne koeficijente korelacije:

– korelacija momenta inercije i modula elastičnosti betona $\rho_{I,Ec} = 0,40$,

– korelacija momenta inercije i krutosti veze $\rho_{I,S\phi} = 0,72$,

– korelacija modula elastičnosti betona i krutosti veze $\rho_{Ec,\,S\phi} = 0,48$,

– korelacija širine i debljine elementa $\rho_{b,\,h} = 0$

Ova preporuka važi za analize pouzdanosti konstrukcija za koje nisu provedena ili su u malom obimu provedena eksperimentalna istraživanja. Kod eksperimentalnih istraživanja većeg obima preporuka može biti konzervativna.

Analiza pouzdanosti, prezentirana u ovom radu, obuhvatila je nepouzdanosti mehaničkih veličina: opterećenja, geometrije i svojstava materijala. Međutim, pouzdanost građevinske konstrukcije zavisi od svih učesnika u izgradnji objekta (investitor, projektant, izvođač i nadzor). Za adekvatnu pouzdanost konstrukcije važne su sve faze «stvaranja» objekta, što podrazumjeva:

– kompetentnost lica ili tima za izradu projektnog zadatka i tenderske dokumentacije;

– odabir projektantske kuće i izvođačke firme sa kvalifikacionom strukturom radnika, opremom i referencama koje smanjuju mogućnost grešaka;

– sistem kontrole kvalitete u pogonima firme koja se bavi proizvodnjom prefabrikovanih konstruktivnih elemenata;

– kompetentnost i stručnost osoblja koje provodi nadzor i kontrolu kvaliteta radova, organizacija i funkcionalnost inspekcijskih službi;

– pogodnost i primjenjivost zakonske regulative iz oblasti osiguranja kvaliteta, prava i obaveza svih učesnika građenja, kaznenih mjera i efikasnosti njihove primjene.

Pored prethodno navedenog, socijalne prilike i stanje morala u lokalnoj ili široj društvenoj zajednici takođe imaju uticaja na pouzdanost objekata. Stoga je za potpuno definisanje prihvatljive pouzdanosti građevinskog objekta neophodno analizirati sve navedene parametre.

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minimum correlation coefficient, is less than the adopted for the experiment conditions. Based on performed analysis is recommended for defining the reliability of the precast connection, which behave similarly as researched precast connections, use the following minimum correlation coefficient:

– correlation between the moment of inertia and modulus of elasticity of concrete

– $\rho_{I,Ec} = 0.40$,

– correlation between the moment of inertia and connection stiffness $\rho_{I,S\phi} = 0.72$,

– correlation between the modulus of elasticity of concrete and connection stiffness

– $\rho_{Ec,\,S\phi} = 0.48$,

– correlation between width and thickness of the element $\rho_{b,\,h} = 0$.

This recommendation applies to the analysis of the reliability of structures that are not implemented or are in small scale experimental research was conducted. With large scale experimental research recommendations may be conservative.

Reliability analysis, presented in this paper, include the unreliability of mechanical quantities: load, geometry and material properties. However, the reliability of civil structures depends on all participants in born of building (investor, designer, contractor and supervision). For adequate reliability of structure all phase of "creating" building are important, which includes:

– competence of the person or team to create a project task and tender documents;

– selection of design houses and contractor with the adequate company qualifications structure of workers, equipment and references, which reduces the possibility of error

– a system of quality control facilities in the company that manufactures prefabricated structures elements;;

– competence and expertise of personnel who supervise and control the quality of work, organization and function of inspection services

– suitability and applicability of the legislation in the field of quality assurance, rights and obligations of all participants of construction, crime rate and efficiency of their application.

In addition to the foregoing, the social conditions and state of moral in the local or wider community also have an impact on the reliability of buildings. Therefore, for the full definition of acceptable reliability of buildings is necessary to analyze all of these parameters.

Note:

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REZIME

POUZDANOST AB MONTAŽNIH VEZA

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U prvom dijelu rada date su teoretske osnove ii pregled stanja aproksimativnih postupaka za analizu pouzdanosti. Nadalje, u radu je prezentiran nastavak istraživanja veze montažne ploče i monolitnog zida, obavljenih u periodu 2004.-2008. godina, koja su provedena sa ciljem definisanja mehanizma rada ovakvih veza. Rezultati istraživanja i preporuke autora koji su provodili slična istraživanja korišteni su za analizu pouzdanosti dobijenih rezultata. Za analizu veze montažne ploče i monolitnog zida korištena je metoda diskretnih tačaka. Na osnovu provedene analize u zaključcima se daju preporuke za analizu sličnih spojeva u građevinskoj nauci i praksi.

Ključne reči: analiza pouzdanosti, funkcija gustoće vjerovatnoće, indeks pouzdanosti, popustljivost, montažna veza, montažna ploča, monolitni zid

SUMMARY

RELIABILITY OF RC PRECAST JOINT

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In the first part of the work given the theoretical basis and review the status of procedures for the approximate analysis of reliability. Furthermore, in the paper was presented continuation of research of prefabricated connection of prefabricated plate and monolithic wall, performed in the period 2004th-2008th years, which were carried out with the aim of defining the work mechanism of such connections. Research results and recommendations of authors who have conducted similar studies were used to analyze the reliability of the obtained results. For analysis of the prefabricated connection of prefabricated plate and monolithic wall was used Point Estimated Method. Based on the analysis in the conclusions are given recommendations for the analysis of similar connections in civil engineering science and practice.

Key words: reliability analysis, probability density function, reliability index, yielding, prefabricated connection, prefabricated plate, monolithic wall